Method Learning

Digital SAT® Math Quick Guide

Use this packet as a quick reference for the most important questions on the Digital SAT Math section.

Format

The Digital SAT Math section consists of two consecutive modules, each with 22 questions presented over 35 minutes.

About 75 percent of questions are four-option multiple choice-format; about 25 percent of questions are free-response.

The section is scored from 200–800.

Finding an Equation to Fit the Data

This topic appears as much as almost anything else on the SAT. The basic structure of this type of problem is this: here are a few (x, y) coordinate pairs; tell us which equation fits (works with) these numbers.

1.

x	$f(x)$
-1	-4
0	-9
1	-14

For the table of values given above, which of the following equations correctly models this data?

(A) $f(x) = -5x - 4$

(B) $f(x) = -5x - 9$

(C) $f(x) = x - 9$

(D) $f(x) = 5x - 9$

2. The points are all on the line $y = 4x + 5$ in only one of the following tables. Which table is it?

(A)

x	y
0	5
2	9
4	13

(B)

x	y
0	5
2	13
4	21

(C)

x	y
0	4
2	9
4	14

(D)

x	y
0	4
2	5
4	6

3.

x	2	5	8
y	4	13	22

Which of the following equations contains each of the three points shown in the above table?

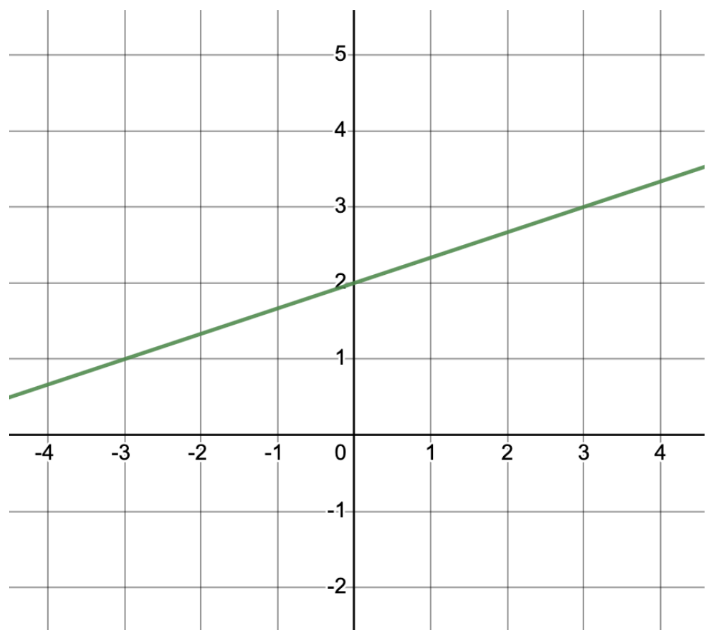
(A) $y = 3x - 2$

(B) $y = -3x + 4$

(C) $y = 3x + 22$

(D) $y = -3x + 9$

4. Which of the following lines is the one shown on this graph in the xy-plane?



(A) $y = \frac{1}{3}x + 2$

(B) $y = \frac{1}{3}x - 2$

(C) $y = -\frac{1}{3}x + 2$

(D) $y = -\frac{1}{3}x - 2$

5. $$f(x) = x^2 + 9x + 14$$

Which of the following points lies on the graph of the quadratic shown above?

(A) $(0, 9)$

(B) $(0, 14)$

(C) $(1, 24)$

(D) both B and C

6. A plumber charges a one-time upfront cost plus an hourly fee. For 3 hours of work, he charges $\$295$, and for 5 hours of work, he charges $\$425$. Which of the following functions models what the plumber charges, $f(x)$, for x hours of work?

(A) $f(x) = 3x + 295$

(B) $f(x) = 130x + 295$

(C) $f(x) = 65x + 295$

(D) $f(x) = 65x + 100$

7.

x	y
-1	5
0	3
1	1

The given table includes three points that are on a linear function that can be written in the form $y = ax + b$. What is the value of $a - b$?

8. For a linear function f, $f(0) = 10$ and $f(4) = 30$. Which of the following equations represents f?

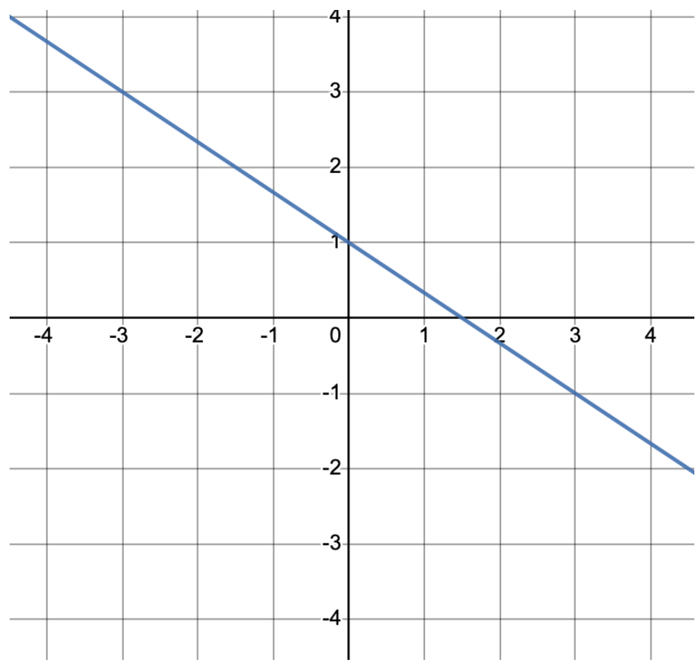
(A) $f(x) = 4x + 30$

(B) $f(x) = 5x + 30$

(C) $f(x) = 4x + 10$

(D) $f(x) = 5x + 10$

9. Which of the following functions is graphed here?



(A) $y = -\frac{2}{3}x - 1$

(B) $y = \frac{2}{3}x - 1$

(C) $y = -\frac{2}{3}x + 1$

(D) $y = \frac{2}{3}x + 1$

Solving Equations

On the SAT, the range of difficulty on problems involving solving equations is pretty high; some of them are really simple problems that could take a few seconds to solve, while some are particularly tricky. The good news is that most of these can be solved using Desmos if you choose to use it.

1. What value of x makes the following equation true?

$$4 + x = 15$$

______

2. If $4x = 108$, what is the value of $3x$?

(A) 27

(B) 36

(C) 81

(D) 135

3. What is $9x$ equal to if we know that $2x = 22$?

______

4. Which of the following equations has the same solution as the equation $3x - 4 = 17$?

(A) $3x = 68$

(B) $3x = 13$

(C) $3x = 21$

(D) $3x = \frac{17}{4}$

5. If $\frac{x}{3} = 11$, what is the value of $\frac{3}{x}$?

______

6. For the function $f(x) = 8x^3$, is the value of x when $f(x) = 216$?

______

7. The function h is defined by $h(x) = 3x - 9$. At an x-value of 7, what is the value of the function, $h(x)$?

______

8. The function g is defined by $g(x) = \frac{1}{2}x - 9$, and $g(n) = 8$, where n is a constant. What is the value of n?

(A) -5

(B) 7

(C) 34

(D) 36

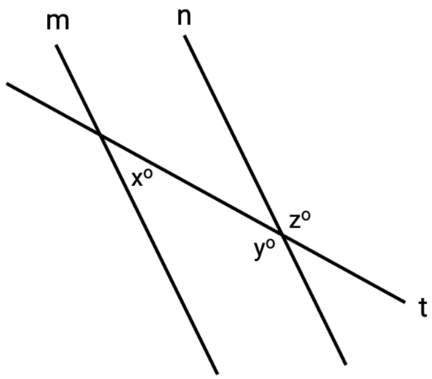
9. If $xy = 3$ and $\frac{36xy}{n} = 27$, what is the value of n?

______

10. If $\frac{x}{y} = 20$ and $\frac{5x}{cy} = 25$, what is the value of c?

____

11. In the picture, m and n are parallel lines. If $x = 3k - 1$ and $y = 10k - 27$, what is k?



Note: figure not drawn to scale

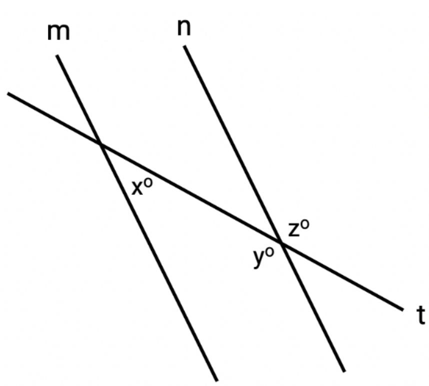
Ⓐ 3

Ⓑ 11

Ⓒ 16

Ⓓ 18

12. This is the same diagram as in the previous problem. What is z?



Note: figure not drawn to scale

____

13. Brian purchases two items at the store and pays a total of $\$138$. The cheaper item costs him x dollars and the more expensive item costs him y dollars, where y is 9 more than double the value of x. How much did the cheaper item cost Brian?

Ⓐ $\$39$

Ⓑ $\$43$

Ⓒ $\$64.50$

Ⓓ $\$95$

14. A rope with a total length of 84 inches is cut into two parts, one with a length of x inches and the other with a length of y inches. If x is 8 less than triple the length of y, what is the value of x?

____

Systems of Equations

This is another popular topic on the SAT. Every time you see a question about a system of equations, you can solve it using Desmos.

1. What is the solution to the following system of equations?

$$2x - 3y = 12$$
$$4x + y = -18$$

(A) $(-6, 3)$

(B) $(-6, -3)$

(C) $(-3, 6)$

(D) $(-3, -6)$

2. At how many points do the graphs of the following equations intersect in the xy-plane?

$$y = x^2 - 8$$
$$y = 2$$

3. How many solutions are there to the following system?

$$y = 8x - 4$$
$$y = -3x - 1$$

(A) zero

(B) one

(C) two

(D) infinitely many

4. In the xy-plane, the solution to the following two equations is the point (x, y). What is x?

$$3x - 2y = 12$$
$$2x + 3y = 21$$

(A) 2

(B) 3

(C) 6

(D) 7

5. The graphs of the equations shown below intersect in the xy-plane at the point (x, y). What is a possible value of x?

$$x^2 - x = 72$$
$$y = -x + 28$$

(A) -9

(B) -8

(C) 19

(D) 36

6.

$$4y = c$$
$$x^2 + 4x + 40$$

In the above system, c is a constant. What is the value of c that makes the system have exactly one intersection point?

7.

$$-x^2 + 4x + 50$$
$$y = 10x + n$$

In the equations above, n is a constant. Which of the following values of n makes the system above have exactly one intersection point in the xy-plane?

(A) -3

(B) 29

(C) 59

(D) 71

---

8. The system of equations shown below has no solution. If c is a constant, what is the value of c?

$$4y - 2x = 10x + 22$$
$$5y - 8 = cx$$

(A) 10

(B) 12

(C) 15

(D) 20

9.

$$\frac{8}{7}x + \frac{11}{23} = 4(cx - n)$$

In the above equation, c and n are constants and $n > 0$. What is the value of c if the equation has no solution?

____

10. Only one of the following systems has infinitely many solutions. Which one?

(A) $y = 4x - 9$
$y = 4x$

(B) $y = 2x + 1$
$y = 2x + 3$

(C) $y = x + 4$
$y = x + 4$

(D) $y = \frac{1}{2}x - 9$
$y = -2x + 3$

---

11. Which of the following systems of equations has exactly one solution?

(A) $y = 5x + 4$
$y = 4x + 5$

(B) $y = 11x - 9$
$y = 11x + 2$

(C) $y = 4x + 1$
$y = 4x + 1$

(D) $y = 6x + 10$
$y = 6x + 11$

Linear Relationships

This is another one of the SAT's favorite topics. Here, we'll see how linear relationships show up on the test.

1. After filling up the gas tank at the gas station, the approximate volume of gasoline (in gallons) left in the gas tank of a particular car is given by the function $f(x) = 18 - 0.04x$, where x is the number of minutes spent driving. What is the best interpretation of the number 18 in this context?

(A) The car uses 18 gallons of gas every minute

(B) The car's gas tank can hold 18 gallons of gas

(C) Every time the driver stops at the gas station, exactly 18 gallons are added to the gas tank

(D) The car travels 18 miles for every gallon of gas it consumes

2. Let's revisit that same scenario: After filling up the gas tank at the gas station, the approximate volume of gasoline (in gallons) left in the gas tank of a particular car is given by the function $f(x) = 18 - 0.04x$, where x is the number of minutes spent driving. What is the best interpretation of the number -0.04 in this context?

(A) For every 0.04 miles driven, the car uses up approximately one gallon of gas

(B) There are 0.04 gallons of gas in the tank when the tank is filled up

(C) For every mile driven, the car uses up approximately 0.04 gallons of gas

(D) The driver adds 0.04 gallons of gas each time he visits the gas station

3. The number of elephants in a population in Africa grew from $9{,}321$ in 2010 to $10{,}959$ in 2019. On average, by how much did the population of elephants grow each year in this time period?

____

4. $$F = \frac{9}{5}x + 32$$

The function F gives the temperature, in degrees Fahrenheit $(^\circ\mathrm{F})$, that corresponds to a temperature of x degrees Celsius $(^\circ\mathrm{C})$. If a temperature increased by $3.6^\circ\mathrm{C}$, by how much did the temperature increase in degrees Fahrenheit?

(A) 1.8

(B) 2

(C) 6.48

(D) 32

5. Rebecca has two jobs: babysitting and tutoring. Last week she made $\$300$ total from x hours babysitting and y hours tutoring. The equation $20x + 30y = 300$ models this situation. How much more per hour does Rebecca make from tutoring compared to babysitting?

(A) $\$10$

(B) $\$15$

(C) $\$30$

(D) $\$50$

6. The number of trees in a certain town in America decreased from $11{,}426$ in 2018 to $10{,}961$ in 2021. On average, by how much did the number of trees decrease each year in this time period?

____

7. The altitude of an airplane in feet as it ascends can be modeled by the function $f(x) = 5{,}000 + 200x$, where x is the number of minutes after the plane's altitude is first measured. Which of the following is the correct interpretation of the number 200 in this context?

(A) the plane ascends by 200 feet every second

(B) the plane's altitude increases by 200 feet each minute

(C) 200 minutes have to pass for the plane to rise $5{,}000$ feet

(D) in $5{,}000$ minutes, the plane's altitude increases by 200 feet

8. Marty bought x pounds of ground beef and y pounds of chicken wings at the grocery store and spent a total of $\$32$. The equation $6x + 4y = 32$ models this scenario. How many more dollars is a pound of ground beef compared to a pound of chicken wings?

(A) $\$2$

(B) $\$4$

(C) $\$6$

(D) $\$8$

9. A freezer company manufactures small freezers and large freezers. For a certain shipment, the company's delivery truck is carrying a total freezer weight of $20{,}000$ pounds. The equation $300x + 500y = 20{,}000$ can be used to model this situation, where x is the number of small freezers in the shipment and y is the number of large freezers. What is the best interpretation of the term $500y$ in this context?

(A) the weight of each large freezer

(B) the number of large freezers in the shipment

(C) the total weight of the large freezers in the shipment

(D) the total weight of the small and large freezers in the shipment

Exponential Functions

There are plenty of functions on the SAT, and if a certain one isn't linear or quadratic, there's a good chance it's exponential—think growing populations of people or investments or the decreasing value of a used car. Here, we'll explore what these questions look like.

1.
$$V(t) = 21{,}000(0.88)^t$$

The exponential equation above is used to determine a car's value V, in dollars, as a function of time t years after it is purchased. What is the meaning of the $21{,}000$ in the equation?

(A) The rate, in dollars per year, at which the car's value changes

(B) The original value of the car when it is purchased

(C) The value of the car one year after it is purchased

(D) The number of days it takes for the car's value to decrease to zero

---

2.
$$V(t) = 21{,}000(0.88)^t$$

The exponential equation above is used to determine a car's value V, in dollars, as a function of time t years after it is purchased. What is the meaning of the 0.88 in the equation?

(A) The car loses 88% of its value per year after it is purchased

(B) The car loses 12% of its value per year after it is purchased

(C) 88% of one year must elapse before the car loses $\$21{,}000$ in value

(D) 88% of one year must elapse before the car loses $\$21{,}000$ in value

---

3 The exponential function f is defined by $f(x) = 7 \cdot a^x$, where a is a positive constant. If $f(4) = 112$, what is the value of $f(6)$?

4. For the function g, the value of $g(x)$ decreases by 25% for every increase in the value of x by 1. If $g(0) = 11$, which equation defines g?

(A) $g(x) = 0.75(11)^x$

(B) $g(x) = 11(0.75)^x$

(C) $g(x) = 1.25(11)^x$

(D) $g(x) = 11(1.25)^x$

---

5. What is the y-intercept of the graph of $g(x) = 3\left(\frac{5}{9}\right)^x$ in the xy-plane?

(A) $(0, 1)$

(B) $(0, 3)$

(C) $(0, 5)$

(D) $(0, 9)$

6.
$$P(t) = 84(1.07)^{\frac{3}{10}t}$$

The function P models the population, in thousands, of a certain city t years after 2011. According to the model, the population is predicted to increase by $c\%$ every 40 months. What is the value of c?

7.

time (in years)	total amount (in dollars)
0	18,000.00
1	16,380.00
2	14,905.80

Deion's car drops in value each year after he purchases it, as shown in the table above. If x is the number of years and $f(x)$ is the value of the car, which of the following functions could represent this situation?

A $f(x) = 18{,}000(0.91)^x$

B $f(x) = 18{,}000(1.09)^x$

C $f(x) = 18{,}000x^{0.91}$

D $f(x) = 18{,}000x^{1.09}$

8.

$$f(x) = 8{,}954(0.71)^{\frac{x}{12}}$$

The function f gives the value, in dollars, of a certain piece of equipment after x months of use. If the value of the equipment decreases each year by $p\%$ of its value the preceding year, what is the value of p?

A 1

B 12

C 29

D 71

9. The amount of money in an investment account decreased by 15% from 2007 to 2008. If the 2008 amount was k times the 2007 amount, what is the value of k?

10. A scientist initially measures $5{,}000$ bacteria in a growth medium. 6 hours later, the scientist measures $10{,}000$ bacteria. Assuming exponential growth, the formula $P = C(2)^{rt}$ gives the number of bacteria in the growth medium, where r and C are constants and P is the number of bacteria t hours after the initial measurement. What is the value of r?

A 6

B $5{,}000$

C $\frac{1}{6}$

D $\frac{1}{5{,}000}$

11. Bacteria are growing in a petri dish. There were $100{,}000$ cells per milliliter during an initial observation. The number of cells per milliliter doubles every 5 hours. How many cells per milliliter will there be 20 hours after the initial observation?

A $400{,}000$

B $1{,}600{,}000$

C $2{,}000{,}000$

D $3{,}200{,}000$

12.

$$g(x) = 5(2)^x$$

The function g is defined by the given equation. If $f(x) = g(x+6)$, which of the following equations defines the function f?

A $f(x) = 11(2)^x$

B $f(x) = 30(2)^x$

C $f(x) = 60(2)^x$

D $f(x) = 320(2)^x$

Quadratic Equations and Functions

If it's not linear and it's not exponential, chances are it's quadratic. You can count on the SAT giving you at least a few questions per test on quadratic equations and functions.

1. Where does the graph of the function $f(x) = x^2 - 9x + 16$ cross the y-axis?

(A) $(0, -9)$

(B) $(0, 16)$

(C) $(-9, 0)$

(D) $(16, 0)$

2. An object is kicked from a platform. The equation $h = -4.9t^2 + 20t + 36$ represents this situation, where h is the height of the object above the ground, in meters, t seconds after it is kicked. Which number represents the height, in meters, from which the object was kicked?

3.

$$g(x) = -3x^2 + 17x + k$$

In the above equation, k is a positive constant. The product of the two solutions to the above quadratic is equal to -13. What must be the value of k?

4. The graph of a parabola in the standard (x, y) plane has x-intercepts at $(-3, 0)$ and $(7, 0)$. Which of the following points could be the vertex of the parabola?

(A) $(-2, -5)$

(B) $(2, -5)$

(C) $(5, -5)$

(D) $(-5, -5)$

5. When the quadratic function f is graphed in the xy-plane, where $y = f(x)$, its vertex is $(-7, 8)$. One of the x-intercepts of this graph is $\left(-\frac{35}{6}, 0\right)$. What is the other x-intercept of the graph?

(A) $\left(-\frac{77}{6}, 0\right)$

(B) $\left(-\frac{49}{6}, 0\right)$

(C) $\left(-\frac{28}{6}, 0\right)$

(D) $\left(\frac{35}{6}, 0\right)$

6. For what value of x does the function $f(x) = 2x^2 - 8x - 17$ reach its minimum?

7. What is the lowest value that the function $f(x) = 2x^2 - 8x - 17$ reaches?

(A) -25

(B) -17

(C) -8

(D) 2

8. Function f is defined by $f(x) = (x - 4)(x - 3)(x + 1)$. Function g is defined by $g(x) = f(x - 3)$. The graph of $y = g(x)$ in the xy-plane has x-intercepts at $(a, 0), (b, 0)$, and $(c, 0)$, where a, b, and c are distinct constants. What is the value of $a + b + c$?

Method Learning

9. The function $f(x) = \frac{1}{6}(x-2)^2 + 8$ gives a ball's height above the ground $f(x)$, in inches, x seconds after it started moving on a track, where $0 \leq x \leq 10$. Which of the following is the best interpretation of the vertex of the graph of $y = f(x)$ in the xy-plane?

A) The ball's minimum height was 8 inches above the ground

B) The ball's minimum height was 2 inches above the ground

C) The ball's height was 8 inches above the ground when it started moving

D) The ball's height was 2 inches above the ground when it started moving

10.
$$49x^2 + bx + 100 = 0$$

In the given equation, b is a constant. For which of the following values of b will the equation have more than one real solution?

A) -126

B) -67

C) 10

D) 168

11. In the xy-plane, a parabola has vertex $(3, -8)$ and intersects the x-axis at two points. If the equation of the parabola is written in the form $y = ax^2 + bx + c$, where $a, b,$ and c are constants, which of the following could be the value of $a + b + c$?

A) -20

B) -12

C) -9

D) -7

12.
$$f(x) = ax^2 + 12x + c$$

In the given quadratic function, a and c are constants. The graph of $y = f(x)$ in the xy-plane is a parabola that opens upward and has a vertex at the point (h, k), where h and k are constants. If $k < 0$ and $f(5) = f(13)$, which of the following must be true?

I. $a \leq 1$

II. $c < 54$

A) both I and II

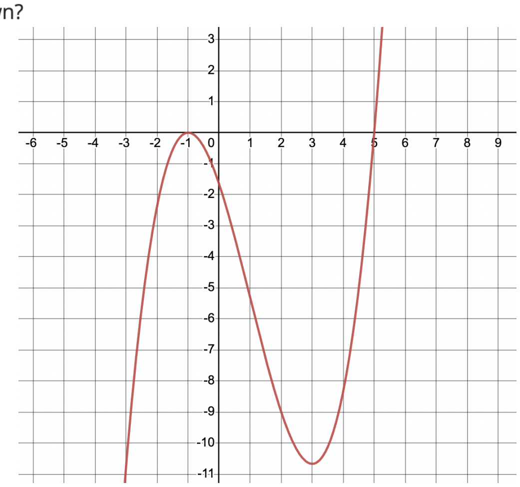
B) neither I nor II

C) I only

D) II only

13. The function f is defined by $f(x) = (x-3)(x-1)(x+5)$. In the xy-plane, the graph of $y = g(x)$ is the result of translating the graph of $y = f(x)$ up 6 units. What is the value of $g(0)$?

14. Which of the following could be the equation of the graph shown?



A) $f(x) = \frac{1}{3}(x-1)^2(x+5)$

B) $f(x) = \frac{1}{3}(x-1)(x+5)$

C) $f(x) = \frac{1}{3}(x-5)(x+1)^2$

D) $f(x) = \frac{1}{3}(x-5)(x+1)$

Method Learning

Finding Intercepts

Here's another topic for which we can work out a lot of the solutions through Desmos. Let's jump in and see what the questions might look like.

1. The function f is defined by $f(x) = (-5)(2)^x + 10$. What is the y-intercept of the graph of $y = f(x)$ in the xy-plane?

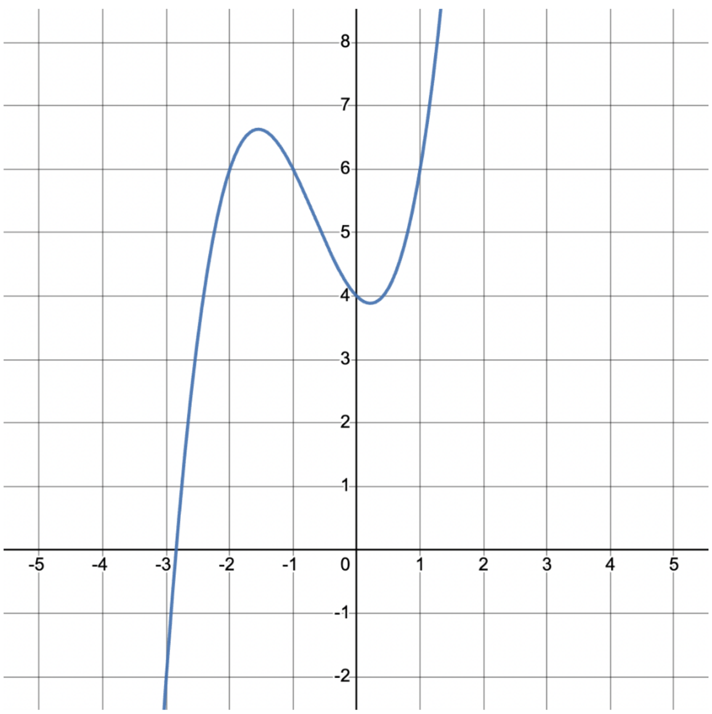
A $(0, 5)$

B $(5, 0)$

C $(0, 1)$

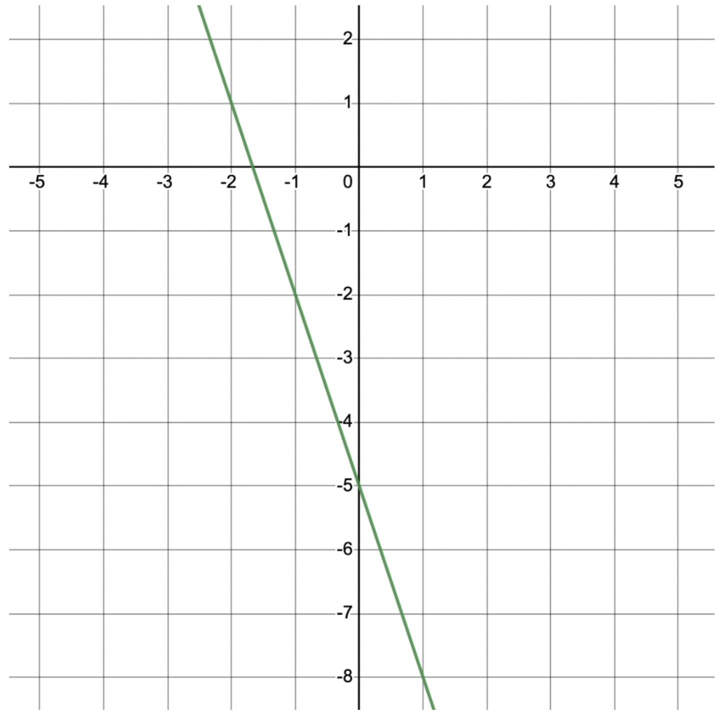
D $(1, 0)$

2. What is the y value of the point where the graph crosses the y-axis?



3. The function h is defined by $h(x) = -6x + 24$. The graph of $y = h(x)$ in the xy-plane has an x-intercept at $(a, 0)$ and a y-intercept at $(0, b)$, where a and b are constants. What is the value of $a + b$?

4. For the function shown, what is $f(0)$?



5. The function f is defined by $f(x) = 2x - 38$. What is the x-intercept of the graph of $y = f(x)$ in the xy-plane?

A $(-19, 0)$

B $(-2, 0)$

C $(2, 0)$

D $(19, 0)$

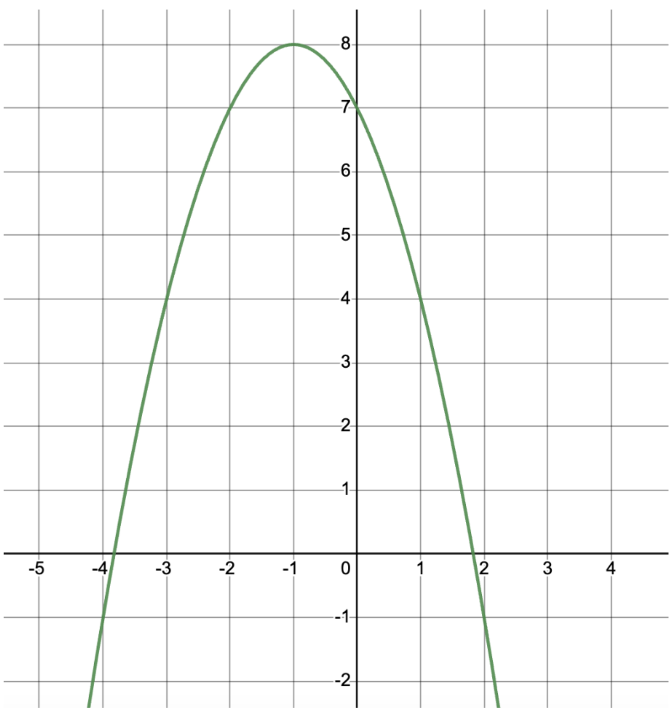
6. The graph of $4x - 9y = 17$ is translated right 1 unit in the xy-plane. What is the x-coordinate of the x-intercept of the resulting graph?

Method Learning

7. The graph of $3x + 7y = 35$ is translated left 6 units in the xy-plane. What is the y-coordinate of the y-intercept of the resulting graph?

8. The points $\left(\frac{17}{2}, 0\right)$ and $\left(0, -\frac{51}{8}\right)$ are the intercepts of the linear function $ax + by = 51$ in the xy-plane, where a and b are integers. What is the value of $a - b$?

9. The y-intercept of the graph shown is $(0, y)$. What is the value of y?



10. When the function $f(x) = -9x + 11$ is graphed in the xy-plane, where $y = f(x)$, what is the value of $f(0)$?

11. The graph of $-11x + 3y = 13$ in the xy-plane has an x-intercept at $(a, 0)$ and a y-intercept at $(0, b)$, where a and b are constants. What is the value of $\frac{a}{b}$?

A) $-\frac{11}{3}$

B) $-\frac{3}{11}$

C) $\frac{3}{11}$

D) $\frac{11}{3}$

12. The graph of $-3x + 8y = 19$ is translated down 2 units in the xy-plane. What is the y-coordinate of the y-intercept of the resulting graph?

13.

x	y
m	14
$m+8$	-10

The table gives the coordinates of two points in the xy-plane. The y-intercept of the line is $(m+1, n)$, where m and n are constants. What is the value of n?

A) 8

B) 9

C) 10

D) 11

Evaluating Functions at a Given Value

Here's where we'll be given a function f(x) and we need to find that function's value at a certain point, like f(-2) or f(8), for example.

1 For the function $y = 4x^2$, what is y when x is 3?

____

2. The function f is defined by $f(x) = 11x + 3$. For what value of x is $f(x) = 25$?

(A) $x = 2$

(B) $x = \frac{28}{11}$

(C) $x = 64$

(D) $x = 278$

3. $$f(x) = 5x - 11$$

Which table gives three values of x and their corresponding values of $f(x)$ for the given function f?

(A)

x	$f(x)$
0	-11
2	-6
4	-1

(B)

x	$f(x)$
0	-11
2	-1
4	9

(C)

x	$f(x)$
0	11
2	16
4	21

(D)

x	$f(x)$
0	-11
2	0
4	5

4. $$f(x) = 3x - 2$$

The function f is defined by the given equation. If $h(x) = f(x-1)$, which of the following equations defines the function h?

(A) $h(x) = 3x - 3$

(B) $h(x) = x - 1$

(C) $h(x) = 2x + 1$

(D) $h(x) = 3x - 5$

5. A car is traveling at a constant speed down a straight portion of road. The function $d = 25t$ gives the distance, in feet from a road marker, that the car will be t seconds after passing the marker. How many feet from the marker will the car be 5 seconds after passing the marker?

(A) 5

(B) 25

(C) 30

(D) 125

6. $$3.5x + 2y = 52$$

The given equation describes the relationship between the number of infants, x, and the number of toddlers, y, that can be cared for at a daycare on a given day. If the daycare has 12 infants on a given day, how many toddlers can it care for on this day?

____

7. The function f is defined by the equation $f(x) = 2^x - 3$. What is the value of $f(4)$?

8. The function f is defined by $f(x) = 11 - 3x$. For what value of x does $f(x) = -13$?

A $\frac{2}{3}$

B 3

C 8

D 50

9. If $f(x) = 3(4)^x$ and $g(x) = f(x+2)$, which of the following functions is g?

A $g(x) = 9(4)^x$

B $g(x) = 6(4)^x$

C $g(x) = 5(6)^x$

D $g(x) = 48(4)^x$

10.

$$f(x) = 2x^2 + 9$$

Which table gives three values of x and their corresponding values of $f(x)$ for the given function f?

A

x	$f(x)$
-1	11
0	9
1	11

B

x	$f(x)$
-1	-11
0	9
1	11

C

x	$f(x)$
-1	-13
0	9
1	13

D

x	$f(x)$
-1	9
0	13
1	21

11. The number of cups of ice cream, $f(x)$ sold in the summer months at an ice cream shop is estimated by the function $f(x) = 8x - 320$, where x is the temperature in degrees Fahrenheit $(^\circ\mathrm{F})$. If the temperature is $75^\circ\mathrm{F}$, approximately how many cups of ice cream will be sold at this ice cream shop?

Finding an Equation to Fit the Data (Answers)

1. **B**

Our first option is to go to Desmos and type in each of the four answers to see which line includes all three points in the table.

Alternatively, we can plug in the first x-value from the table, -1, into each of the equations in the answers until we get an equation that gives us the correct y-value from the table, -4.

If we happen to find that two equations work for this pair, $(-1, -4)$, then we can move to the next x-value, 0, and repeat this process until we find that only one equation works, which will be B.

---

2. **B**

We have a few options here: type the line into Desmos and check which table's points are all on the line (this shouldn't be too hard).

Or we can notice that each table's x-values are $2, 4,$ and 6, which means we can plug each of these into the function to see what y-value must go with each of these x-values. We'll do that method here, but feel free to use Desmos to get the answer.

$\text{test } x = 0$	$\text{test } x = 2$	$\text{test } x = 4$
$y = 4x + 5$	$y = 4x + 5$	$y = 4x + 5$
$y = 4(0) + 5$	$y = 4(2) + 5$	$y = 4(4) + 5$
$y = 0 + 5$	$y = 8 + 5$	$y = 16 + 5$
$y = 5$	$y = 13$	$y = 21$
$(0, 5)$	$(2, 13)$	$(4, 21)$

So the answer is B.

---

3. **A**

Our first option is to go to Desmos and type in each of the four answers to see which line includes all three points in the table.

Alternatively, we can plug in the first x-value from the table, 2, into each of the equations in the answers until we get an equation that gives us the correct y-value from the table, 4.

If we happen to find that two equations work for this pair, $(2, 4)$, then we can move to the next x-value, 5, and repeat this process until we find that only one equation works, which will be A.

4. **A**

The line goes up, so the slope is positive.

The line passes through positive 2 on the y-axis, so the y-intercept is positive.

The only line with both of these values positive is the answer: A.

---

5. **D**

We can type the function into Desmos to check each point in the answer. Alternatively, we could plug in each x-value from the answers to see its corresponding y-value. Either way, we will see that both $(0, 14)$ and $(1, 24)$ are on this quadratic's graph. Be careful of answer options like D that include more than one answer!

---

6. **D**

From the problem, we can find two points that this function passes through: $(3, 295)$ and $(5, 425)$.

So we know that we can plug these points into the answers until we find one that works with both of these points. D does:

$f(x) = 65x + 100$	$f(x) = 65x + 100$
$295 \stackrel{?}{=} 65(3) + 100$	$425 \stackrel{?}{=} 65(5) + 100$
$295 \stackrel{?}{=} 195 + 100$	$425 \stackrel{?}{=} 325 + 100$
$295 = 295$ ✓	$425 = 425$ ✓

---

7. **-5**

The easy part to find is this line's y-intercept; the fact that the point $(0, 3)$ is on the line tells us the y-intercept, meaning the b in our equation, is 3:

$$b = 3$$

The slope will be slightly more difficult. Each time the x-value in the table goes up by 1, the y-value goes down by 2. This means the slope is -2.

$$a = -2$$

solution is continued on the next page...

If you don't like that slope method, remember that you can pick two points from the table, calling the first one (x_1, y_1) and the second one (x_2, y_2). Then you can use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. Let's use $(0, 3)$ and $(1, 1)$:

$$\begin{aligned} m &= \frac{1-3}{1-0} \\ &= \frac{-2}{1} \\ &= -2 \end{aligned}$$

Now we find $a - b$:

$$\begin{aligned} &a - b \\ &= -2 - 3 \\ &= -5 \end{aligned}$$

---

8. **D**

$f(0) = 10$ means that this function goes through the point $(0, 10)$, and since the x-coordinate is 0, this means 10 is the y-intercept.

So the line in $y = mx + b$ form ends in 10: $y = mx + 10$.

This eliminates A and B.

$(0, 10)$ is the first point on the line and $(4, 30)$ is the second. Let's use the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slope between these two points, which we'll call (x_1, y_1) and (x_2, y_2):

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{30 - 10}{4 - 0} \\ &= \frac{20}{4} \\ &= 5 \end{aligned}$$

So the line is $y = 5x + 10$.

9. **C**

The slope is negative because the line is going down from left to right. The y-intercept is positive since the line crosses the y-axis above the origin, at the point $(0, 1)$.

The only line in the answers with a negative slope and a positive y-intercept must be our answer. It's C.

Solving Equations (Answers)

1. **11**

$$\cancel{4} + x = 15$$
$$\cancel{-4} \qquad -4$$
$$\boxed{x = 11}$$

2. **C**

Since $4x = 108$, we divide both sides by 4 to find that $x = 27$. Since we need $3x$, we do $3(27)$ to get our answer: 81.

3. **99**

Since $2x = 22$, $x = 11$. This means that $9x = 9(11) = 99$.

4. **C**

We'll solve the equation $3x - 4 = 17$ for $3x$ and we get $3x = 21$. That's the answer!

5. **1/11**

They flipped the left side of the equation to get $\frac{3}{x}$, so we just need to flip the right side (you can make it a fraction first by putting it over 1 if you'd like), and we get our answer: $\frac{1}{11}$.

6. **3**

Let's plug in 216 for $f(x)$ in the equation $f(x) = 8x^3$, then solve it for x:

$$f(x) = 8x^3$$
$$216 = 8x^3$$
$$27 = x^3$$
$$\boxed{3 = x}$$

7. **12**

They tell us that $x = 7$ and so we just plug 7 in for x in the function:

$$h(x) = 3x - 9$$
$$h(x) = 3(7) - 9$$
$$h(x) = 21 - 9$$
$$\boxed{h(x) = 12}$$

8. **C**

n is the value that we plug into the function to get out a value of 8. So we can solve this problem by plugging in 8 in place of $f(x)$ and solving for x. The value we get for x will be our n value:

$$g(x) = \frac{1}{2}x - 9$$
$$8 = \frac{1}{2}x - 9$$
$$17 = \frac{1}{2}x$$
$$34 = x$$

Therefore, the value of n is 34.

Note: to go from $17 = \frac{1}{2}x$ to the answer, we could either divide by $\frac{1}{2}$ on both sides or multiply both sides by 2—both of these methods are equivalent to each other. That's how we get x to be 34.

9. **4**

Since $xy = 3$, we can plug 3 into the other equation in place of xy:

$$\frac{36xy}{n} = 27$$
$$\frac{36(3)}{n} = 27$$
$$\frac{108}{n} = 27$$
$$108 = 27n$$
$$\frac{108}{27} = n$$
$$\boxed{n = 4}$$

10. **4**

We can solve $\frac{x}{y} = 20$ for x to get $x = 20y$, then we'll substitute this into the other equation in place of x:

$$\frac{5x}{cy} = 25$$

$$\frac{5 \cdot 20\cancel{y}}{c\cancel{y}} = 25$$

$$\frac{100}{c} = 25$$

$$100 = 25c$$

$$\frac{100}{25} = c$$

$$\boxed{c = 4}$$

---

11. **C**

x and y are supplementary because they are same-side interior angles. Therefore, we can add them together and set their sum equal to 180° and solve for k:

$$x + y = 180$$
$$3k - 1 + 10k - 27 = 180$$
$$13k - 28 = 180$$
$$13k = 208$$
$$k = \frac{208}{13}$$
$$\boxed{k = 16}$$

---

12. **133**

Because we know that $k = 16$, we can find the measure of y by plugging 16 in place of k in the expression for y:

$$y = 10k - 27$$
$$y = 10(16) - 27$$
$$y = 160 - 27$$
$$y = 133$$

Since the measure of y is 133°, z must also be 133° because y and z are vertical angles, which are always congruent.

13. **B**

We could set this one up as follows:

$$x + y = 138$$
$$y = 2x + 9$$

We didn't mention this in this lesson, but at this point, we could also solve this by typing this system into Desmos. The intersection point (x, y) is what we'd be looking for. That point is $(43, 95)$, so the cost of the cheaper item, the value of x, is $\$43$.

If we did it algebraically, we'd get

$$x + (2x + 9) = 138$$
$$3x + 9 = 138$$
$$3x = 129$$
$$x = 43$$

---

14. **61**

The important thing to realize here is that the total length, 84, is made up of the length of x plus the length of y. In other words,

$$x + y = 84$$

The next thing to notice is that we can write an expression for x in terms of y. They tell us that x is 8 less than triple the length of y. This translates into

$$x = 3y - 8$$

Remember, 8 less than a number is that number minus 8.

Since we now have an expression for x in terms of y, let's plug that into the first equation and solve it for y:

$$x + y = 84$$
$$3y - 8 + y = 84$$
$$4y - 8 = 84$$
$$4y = 92$$
$$\boxed{y = 23}$$

But we need to find x, not y, so we can plug this into the equation $x = 3y - 8$ to solve for x:

$$x = 3y - 8$$
$$x = 3(23) - 8$$
$$x = 69 - 8$$
$$\boxed{x = 61}$$

Systems of Equations (Answers)

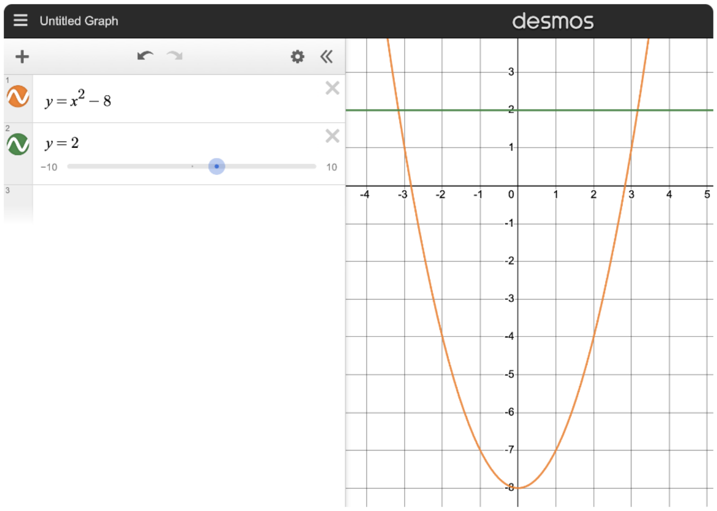
1. **D**

Go to Desmos, type in the equations, and see that the solution is $(-3, -6)$.

---

2. **2**

Here, we can simply find from the xy-plane how many times the two graphs intersect. This should be simple!



In Desmos, we see that these graphs intersect at two points. So our answer is 2.

There are other ways they can phrase this same question, including *how many solutions does the following system have?* or *the following system has how many intersection points?* Just be aware that these all mean the exact same thing.

---

3. **B**

We can easily see in Desmos that the graphs of these equations intersect in only one point.

---

4. **C**

In Desmos, we can see that the intersection point is clearly $(6, 3)$. Thus, the answer is C.

---

5. **B**

Type in both equations to Desmos and see where the two graphs intersect. We'll see that the two intersection points are $(-8, 36)$ and $(9, 19)$. So the *possible* x-values are -8 and 9, but only one of these is listed as an answer choice: -8. So our answer is B.

6. **144**

Notice that the second equation represents a quadratic. If you type this into Desmos, you'll see that the vertex—the very bottom point on this particular parabola—has coordinates $(-2, 36)$.

The other graph, $4y = c$, represents a horizontal line, since any equation with a y but no x is a horizontal line. In order for these two graphs to have a single point of intersection, that intersection point *must* be the vertex—if you think about it, you'll realize that there are no other possibilities.

So what does the value of c need to be? Let's go to Desmos and type in the first equation, if you haven't already. The problem is that Desmos doesn't show anything for the graph of $4y = c$. Desmos would prefer for this equation to have an x-term in it, but we just can't change the equation up, right? So what do we do?

We can write the equation like this: $4y = 0x + c$. Notice that this is the same thing as the original equation since $0x$ is just 0, and adding this to c is still c.

So if we type in this, we'll see that Desmos gives us the option to add a slider for c. Let's do this (see our videos on using Desmos if you aren't sure of how to use the slider feature).

Change the range of the slider so that it allows for larger numbers, and you'll see that the value for c that works here is 144.

---

7. **C**

We can see in Desmos that the second graph doesn't represent a horizontal line, as we saw earlier, but rather a line that is slanted. We can add the slider and play around with it until we see that the line intersects the parabola at just one point. If we go close to where this intersection point is and click on the graph, Desmos should automatically show us what that point of intersection is: $(-3, 29)$.

But be careful! They ask for the value of our constant, n, *not* either of these numbers from our coordinates. The value of n (from our slider) is 59.

---

8. **C**

Let's use Desmos again. Remember, *no solution* means *no intersection point*, and for this to happen, <u>the lines must be parallel</u>. So that's what we'll be looking for.

If we type both equations into Desmos, we can add a slider for c. The value for c that makes the lines parallel is 15.

9. **2/7**

We can set both of the coefficients of x equal to each other since there's no solution and therefore the lines must be parallel (same slope).

First, we should probably distribute on the right side of the equation:

$$\frac{8}{7}x + \frac{11}{23} = 4cx - 4n$$

Now we'll set those coefficients equal:

$$\frac{8}{7} = 4c$$

$$\frac{8}{7 \cdot 4} = c$$

$$\boxed{c = \frac{2}{7}}$$

---

10. **C**

Infinitely many solutions means infinitely many intersection points, which means that the lines overlap everywhere and are therefore the exact same line. The only way this can happen is if <u>the equations are exactly the same</u>. The answer that this happens in is C.

11. **A**

Exactly one solution means that the lines intersect once and then go their separate ways. This is the normal case that we see most often. It occurs simply when <u>the slopes of the lines are different</u>.

In slope-intercept form, $y = mx + b$, the slope is the number in front of x. Here, the slopes are different in only one answer: A.

Linear Relationships (Answers)

1. **B**

The number 18 is in the position of the y-intercept in this equation (think $y = mx + b$, but this time, the equation is simply written backward: $y = b + mx$). And we know that the y-intercept is the value of the function when $x = 0$. Since x represents miles driven, if $x = 0$, then no miles have been driven after stopping at the gas station to fill up the tank.

This means that there must be 18 gallons of gas in the tank after filling it up. The answer is B.

---

2. **C**

For the same reason as in the last problem, we know that -0.04 is the slope of this equation. The slope is always the change in y for every 1-unit change in x. Putting it into the context of our problem, y, the amount of gas left in the tank, *decreases* by 0.04 gallons (that's why it's -0.04) for every additional mile that the car is driven.

This is what the answer in C says, making that our answer.

---

3. **182**

The population grew by a total of $1{,}638$ elephants in this time period, since $10{,}959 - 9{,}321 = 1{,}638$.

To find the average increase per year, we can divide this number by how many years passed between the starting year, 2010, and the ending year, 2019: 9 years.

$$\frac{1{,}638}{9} = 182$$

---

4. **C**

The best way to do this is probably to choose two Celsius temperatures that differ by $3.6^\circ\mathrm{C}$ (this meets the criteria of "a temperature increased by $3.6^\circ\mathrm{C}$).

Two good ones to choose are $0^\circ\mathrm{C}$ and $3.6^\circ\mathrm{C}$. We choose 0 since it makes our math easier. Then we can convert them to Fahrenheit and compare by how much they differ in $^\circ\mathrm{F}$ and that difference will be our final answer:

$$\begin{array}{cc} 0^\circ\mathrm{C} \rightarrow x = 0 & 3.6^\circ\mathrm{C} \rightarrow x = 3.6 \\ F = \frac{9}{5}x + 32 & F = \frac{9}{5}x + 32 \\ F = \frac{9}{5}(0) + 32 & F = \frac{9}{5}(3.6) + 32 \\ F = 0 + 32 & F = 6.48 + 32 \\ F = 32^\circ & F = 38.48^\circ \end{array}$$

38.48° is 6.48° more than 32°, so C is our answer.

5. **A**

This one is pretty easy: Rebecca earns $\$20$ per hour babysitting and $\$30$ per hour tutoring. So she makes $\$10$ more per hour tutoring, and that's our final answer.

---

6. **155**

The number of trees decreased by a total of 465 trees in this time period, since $10{,}961 - 11{,}426 = -465$.

To find the average decrease per year, we can divide this number by how many years passed between the starting year, 2018, and the ending year, 2021: 3 years.

$$\frac{465}{3} = 155$$

Note: we can disregard the negative sign in our calculations since all that the sign tells us is that the number dropped, which we already knew from the language of the problem.

---

7. **B**

We can plug in successive x-values into the function to see what happens to the altitude every minute. If $x = 0$, the altitude is $5{,}000$ feet. If $x = 1$, the altitude is $5{,}200$ feet. If $x = 2$, the altitude is $5{,}400$ feet, etc.

So it's clear that the plane rises in altitude by 200 feet each minute.

---

8. **A**

Let's talk first about the fact that the 32 in the equation is the dollar amount that they gave us in the problem: $\$32$ spent at the store. Therefore, in order to keep our units consistent, $6x$ and $4y$ must also be dollar amounts. Since x is pounds of ground beef and y is pounds of chicken wings, $6x$ is the *total* cost of the ground beef and $4y$ is the *total* cost of the chicken wings.

This means that *each* pound of ground beef costs $\$6$ and *each* pound of chicken wings costs $\$4$, so to answer the question, we need the difference between these two numbers: $\$6 - \$4 = \$2$. A pound of ground beef is $\$2$ more than a pound of chicken wings.

As we've said before, when comparing coefficients like this, the shortcut will typically just be to subtract the two given coefficients. If we'd done this from the start, we'd have gotten our answer really quickly.

9. **C**

This time, we just need to tell them what the term $500y$ refers to. Think about it: this equation has to do with the shipment's total weight in pounds, and because $500y$ goes with the y term, the number of large freezers in the shipment, it makes sense that $500y$ would be the total weight of all of the large freezers in the shipment. And that's it!

Note: had they asked for the meaning of the number 500, that would be the weight in pounds of *each* large freezer, whereas $500y$ is the total weight of *all* the large freezers combined.

Exponential Functions (Answers)

1. **B**

Remember that the constant term out front within an exponential equation always represents the y-intercept of the equation on a graph. When time is the x-variable (as it is here), this means the constant represents the "starting" value of the function, where $t = 0$. Therefore, the $21{,}000$ represents the original value of the car when it is purchased (that is, 0 years after it is purchased).

---

2. **B**

The number raised to an exponent always indicates the growth or shrink of the function.

If the number is greater than 1, this represents growth equivalent to an amount greater than 1 added on. For example, if the number is 1.32, the number would represent 32% growth.

If the number is less than 1, this represents a shrink equivalent to the amount less than 1. The 0.88 must have come from $(1 - 0.12)$: this represents a decrease of 12% of the car's value each year.

---

3. **448**

Notice that they tell us $f(4) = 112$. This means that plugging in 4 for x gives us an answer of 112. Let's use this information to find a:

$$f(x) = 7 \cdot a^x$$

$$112 = 7 \cdot a^4$$

$$\frac{112}{7} = a^4$$

$$16 = a^4$$

$$a = 2$$

Now we can take this value and plug it into the function for a:

$$f(x) = 7 \cdot 2^x$$

Finally, we can plug in 6 for x to find the value of $f(6)$:

$$\begin{aligned} f(6) &= 7 \cdot 2^6 \\ &= 7 \cdot 64 \\ &= 448 \end{aligned}$$

4. **B**

"Decreases by 25% for every increase in the value of x by 1" just means that we're dealing with an exponential function with a parentheses value of 0.75.

$g(0) = 11$ means that the initial value of the function, the number in front of the parentheses, is 11. Remember, this is the value of g when $x = 0$, which is what $g(0) = 11$ means.

Putting this all together, our answer must be $g(x) = 11(0.75)^x$.

---

5. **B**

The y-intercept always occurs when $x = 0$. So all we need to do here is plug in 0 in place of x in the equation and see what that gives us:

$$g(0) = 3\left(\frac{5}{9}\right)^0$$

$$g(0) = 3 \cdot 1$$

$$g(0) = 3$$

So the point $(0, 3)$ is the answer.

---

6. **7**

Let's convert 40 months to a fraction to see what's going on:

$$40 \text{ months} \cdot \frac{1 \text{ year}}{12 \text{ months}}$$

$$= \frac{40}{12} \text{ years}$$

$$= \frac{10}{3} \text{ years}$$

Once again, this is the reciprocal of the fraction in the exponent of the original function. So if we plug this in for t, since it's a number of years, we get

$$84(1.07)^{\frac{3}{10} \cdot \frac{10}{3}}$$

But multiplying a fraction by its reciprocal always gives us 1, and raising something to the first power doesn't change its value. So we get

$$84(1.07)$$

They ask us by how much the population will increase, and multiplying by 1.07 represents a 7% increase.

7. **A**

Notice that only A and B follow the correct format for an exponential function (the variable as the exponent), so it's got to be one of those answers.

Also, notice that the value of the car is dropping, so the number in the parentheses must be a number less than 1, leaving us with our answer: A.

---

8. **C**

We can actually convert the units of our original function from months to years. Since 12 months is 1 year, the function would be the same if we changed it to

$$f(x) = 8{,}954(0.71)^t$$

where t is the time in years. Don't know why? Because 12 months is 1 year, plugging in 12 into the original function for m should give us the same result as plugging in 1 for t in the rewritten function:

$$f(x) = 8{,}954(0.71)^{\frac{x}{12}} \qquad f(x) = 8{,}954(0.71)^t$$
$$f(x) = 8{,}954(0.71)^{\frac{12}{12}} \qquad f(x) = 8{,}954(0.71)^1$$

Since $\frac{12}{12} = 1$, these two *do* have the same value.

Now for our answer: how much value does this function lose each year? Now it's just a matter of figuring out what percent 0.71 corresponds to:

$$\begin{aligned} &0.71 \\ &= 1.00 - 0.29 \\ &= 100\% - 29\% \end{aligned}$$

So we get a decrease of 29% each year, and 29 is the answer.

---

9. **0.85**

We have to be a little careful this time because the value is decreasing. This means we'll need to multiply by a number less than 1. Since it decreases by 15%, here's the math we can do to work out our answer:

$$\begin{aligned} &100\% - 15\% \\ &= 85\% \\ &= 0.85 \end{aligned}$$

10. **C**

Notice that the number in parentheses is 2, which corresponds to a doubling of the population. Notice also that the population that they give us after 6 hours is exactly *double* the original population. Since this occurs at $t = 6$, this must be the value of t in our function:

$$P = C(2)^{6r}$$

And because we know that the population must double after 6 hours, we need the exponent to be 1, because $2^1 = 2$ and any other exponent would no longer correspond to a doubling. In other words, $6r = 1$.

Now we can solve this for r:

$$6r = 1$$
$$r = \frac{1}{6}$$

---

11. **B**

If the population doubles every 5 hours, then 20 total hours corresponds to 4 doublings: $\frac{20}{5} = 4$.

We can simply double $100{,}000$ four times to get our answer: $1{,}600{,}000$.

Alternatively, we could recognize that this problem corresponds to the function $100{,}000 \cdot 2^4$, and plugging this into the calculator gives us the same result.

---

12. **D**

The fact that $f(x) = g(x+6)$ means we can find f by plugging $x + 6$ into the function g in place of x:

$$f(x) = 5(2)^{x+6}$$

Now we need to rewrite this using the rules of exponents:

$$\begin{aligned} f(x) &= 5(2)^{x+6} \\ &= 5(2)^x(2)^6 \\ &= 5(2)^x \cdot 64 \\ &= 64 \cdot 5(2)^x \\ &= 320(2)^x \end{aligned}$$

Quadratic Equations and Functions (Answers)

1. **B**

The place where the graph crosses the y-axis is called the y-intercept, and this is always the constant at the end of a quadratic that's written in the form that ours is written in here.

So we definitely know 16 is part of the answer. But why is $(16, 0)$ wrong? That point is on the x-axis, not the y-axis.

---

2. **36**

The *height from which the object is kicked* refers to the starting point of the object; in other words, the height of the ball when the time, t, is 0. So we just need to plug in 0 into the function to find the answer.

Doing this, we see that the only number left standing is the constant at the end, 36. And this is our answer!

---

3. **39**

This might not have been something you learned about in school, but for any quadratic function of the form $ax^2 + bx + c$, the sum of the two solutions is always equal to $-\frac{b}{a}$ and the product of the two solutions is always equal to $\frac{c}{a}$. We won't talk about why this is the case, but it's just something to try to remember, as anytime they ask us for the sum or the product of the two solutions to a quadratic, knowing these are typically much faster than other methods.

Since the product of the solutions for any quadratic must be equal to $\frac{c}{a}$, and since they tell us this needs to be -13 in this particular problem, we can set these two things equal:

$$\frac{c}{a} = -13$$

But we know that the value of a, the leading coefficient, is -3, and c is really the constant k that we are looking for, so we can plug these into the above equation and solve it for k:

$$\frac{k}{-3} = -13$$

$$k = -13(-3)$$

$$\boxed{k = 39}$$

---

4. **B**

There are many ways to solve this problem, but the easiest is to use your understanding of the relationship between the zeros (x-intercepts) of a parabola and its vertex. The x-coordinate of the vertex must be exactly halfway between the x-coordinates of the zeros of the parabola. Since we have those, we can simply find the midpoint of the zeros by taking their average:

$$\frac{-3+7}{2} = \frac{4}{2} = 2$$

The vertex, therefore, *must* be located at point $(2, ?)$. The only option with $x = 2$ is choice B.

---

5. **B**

Like in the last problem, we know that the vertex's x-value must be the average of the x-values of the two x-intercepts. Let's just call our missing x-intercept c so we can solve for it in the following equation, where the left side is the average of the two x-intercepts and the right side is the vertex's x-value:

$$\frac{-\frac{35}{6} + c}{2} = -7$$

$$-\frac{35}{6} + c = -14 \qquad \text{multiply both sides by 2}$$

$$-\frac{35}{6} + c = -\frac{84}{6} \qquad \text{make -14 a fraction over 6}$$

$$c = -\frac{84}{6} + \frac{35}{6} \qquad \text{add } \tfrac{35}{6} \text{ to both sides}$$

$$\boxed{c = -\frac{49}{6}}$$

---

6. **2**

The language of this problem (*minimum*) suggests we want to find something related to the vertex. When they say "for what value of x," that's our key to simply find the x-value of the vertex.

The way to do this is using $x = \frac{-b}{2a}$:

$$f(x) = 2x^2 - 8x - 17$$

$$a = 2,\ b = -8,\ c = -17$$

$$x = \frac{-b}{2a}$$

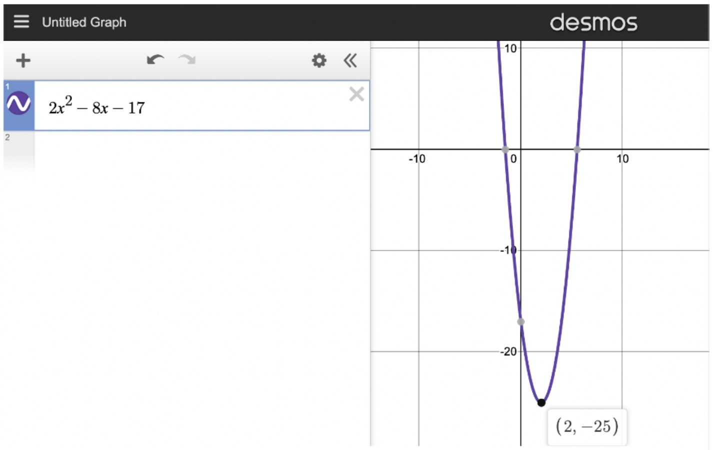
$$x = \frac{-(-8)}{2(2)}$$

$$x = \frac{8}{4}$$

$$\boxed{x = 2}$$

solution is continued on the next page...

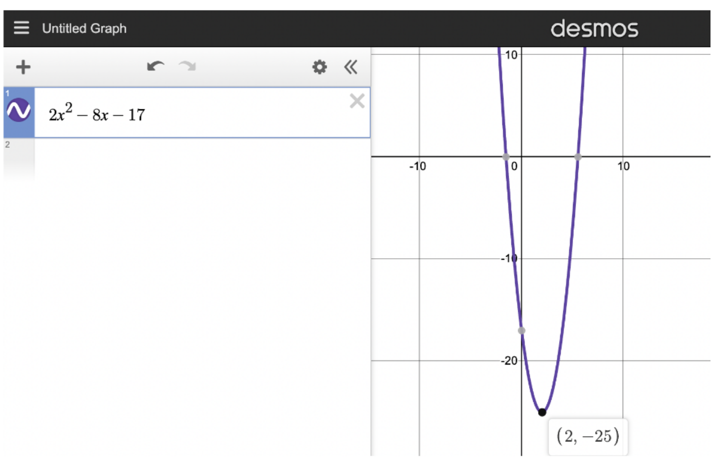
A simple check through Desmos would show that this is the answer if you wanted to avoid all of the above work!



7. **A**

Notice the subtle difference between the last problem and this one; here we need to find the *lowest value that the function reaches*. In other words, this time we want the y-value of the vertex.

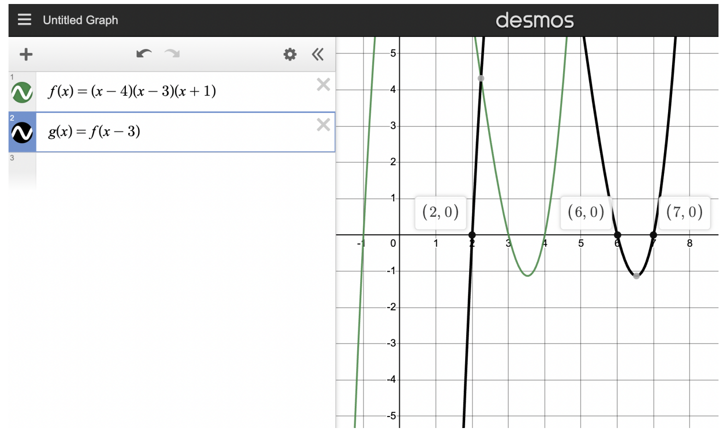
So we could either take the answer from the last problem, the x-value of the vertex, and plug it into the function to find the corresponding y-value, or we could simply check Desmos to see the lowest point on the function. Either way, we get -25.



8. **15**

This is easy if we know what to look for in Desmos; we just need to add up the three x-values of g's three x-intercepts. Desmos will show us that these three values are $2, 6,$ and 7, and $2 + 6 + 7 = 15$. Easy!

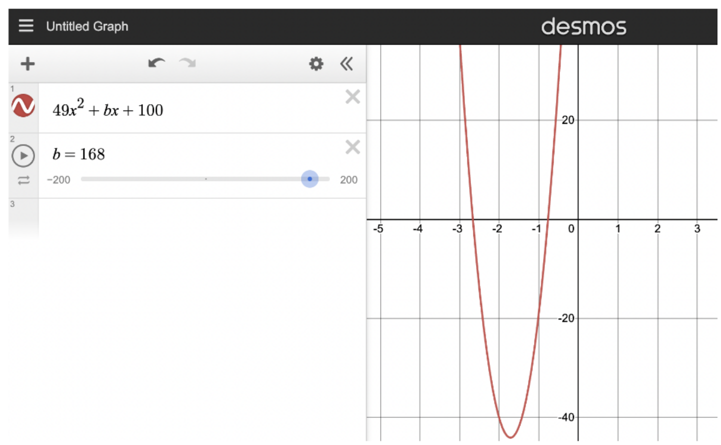
Note: Technically this is a cubic function since it has three roots, but we included it in our lesson on quadratics since the way to solve it is so related to everything we've been learning here.



9. **A**

The vertex of the parabola here would be $(2, 8)$. Because the leading coefficient is positive, the vertex is a *minimum* for this parabola. So at a time of 2 seconds, the height of the ball is 8 inches, and this is the lowest (aka minimum) that the ball ever reaches. Therefore, the answer is A.

10. **D**



We could definitely use the discriminant here, but the easier way, especially with such large numbers in the answers, would just be to go to Desmos and type in $y = 49x^2 + bx + 100$ as our equation. Then Desmos will offer us the chance to add a slider for b, which we'll want to do.

None of the default values for b between -10 and 10 work. Why not? Because the problem says that we want *more than one real solution*, which means more than one x-intercept. So we want our graph to cross the x-axis twice.

Let's make b go from -200 to 200, just based on our answer choices, and doing so shows us that the only b in the answers that gives us two x-intercepts is 168, our answer.

11. **D**

This is one of the hardest problems we'll see. They tell us that the vertex is $(3, -8)$ and that the parabola intersects the x-axis at two points. Based on where the point $(3, -8)$ is, this means the parabola has to open upward, and so a, our leading coefficient, must be positive: $a > 0$.

We could also start to write this parabola's equation in vertex form since we know the coordinates of the vertex:

$$y = a(x-h)^2 + k \qquad \text{vertex form (in general)}$$
$$y = a(x-3)^2 - 8 \qquad \text{vertex form (in our example)}$$

FOILing and combining like terms, we get

$$y = ax^2 - 6ax + 9a - 8$$

So our a, b, and c values are as follows:

$$a = a$$
$$b = -6a$$
$$c = 9a - 8$$

So the sum of these three, $a + b + c$, is

$$a + b + c$$
$$= a + (-6a) + (9a - 8)$$
$$= 4a - 8$$

Since this is what the sum in question is equal to, we could test out each of the four answers by setting each equal to this expression, $4a - 8$, solving each for a, and considering if that answer choice yields a possible a-value, knowing that, as we said earlier, a has to be positive:

If we test A, we get

$$4a - 8 = -20$$
$$4a = -12$$
$$a = -3$$

But a has to be positive, so this is wrong. Similarly, B and C are wrong. This leaves us with our answer, D. Let's see why it's correct:

$$4a - 8 = -7$$
$$4a = 1$$
$$a = \frac{1}{4}$$

And this works, since all we care about is the fact that a is positive. So our answer is D.

12. **A**

This is one of the hardest questions that we'll see. First, notice that they tell us the parabola opens upward. This means that our a, the leading coefficient, has to be positive.

They also tell us that $f(5) = f(13)$. How is this helpful? Remember that parabolas are symmetric, and the vertex is always on the line of symmetry. So if $f(5) = f(13)$, then the x-value of the vertex must be the number right between 5 and 13, which is their average, 9. But this is the same as h in the coordinates of the vertex, (h, k). So $h = 9$.

What else do we know? We've learned that the x-coordinate of the vertex of any quadratic is always $x = -\frac{b}{2a}$, which means that $-\frac{b}{2a} = 9$. And because we know b is -12, the coefficient of x, this equation becomes

$$-\frac{-12}{2a} = 9$$

and we can solve this for a to get

$$a = \frac{12}{18}$$

which reduces to

$$a = \frac{2}{3}$$

So we know that I is correct.

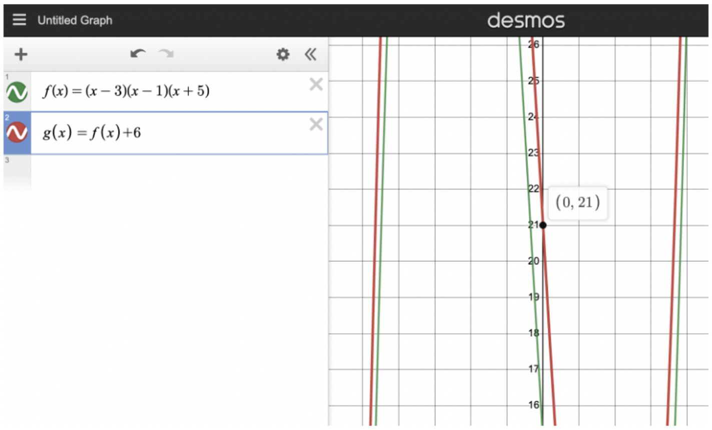
What else can we do? They also tell us that $k < 0$, so since h is positive and k is negative, the vertex must be below the x-axis down in the fourth quadrant. All this means is that, since the parabola opens upward, there will be two real roots (solutions) to this quadratic; this means that the discriminant is positive: $b^2 - 4ac > 0$. *This is the key to the rest of this problem*!

Let's plug what we know into this inequality:

$$b^2 - 4ac > 0$$
$$(-12)^2 - 4\left(\frac{2}{3}\right)c > 0 \qquad \text{substitute}$$
$$144 - \frac{8}{3}c > 0 \qquad \text{simplify}$$
$$144 > \frac{8}{3}c \qquad \text{move } c \text{ term to the right side}$$
$$\frac{8}{3}c < 144 \qquad \text{flip the inequality around}$$
$$c < 144 \cdot \frac{3}{8} \qquad \text{multiply by the reciprocal on both sides}$$
$$c < 54 \qquad \text{simplify}$$

So I and II are both true, and the answer is A.

13. **21**



First off, finding $g(0)$ is equivalent to finding g's y-intercept, since an x-value of 0 always occurs at a graph's y-intercept. So that's what we're looking for.

Now we can use Desmos to see the graph of $f(x)$, and from there, we can simply shift that up by 6 to figure out where $g(x)$ would cross the y-axis.

Alternatively, it's helpful to know that a vertical shift up 6 is equivalent to saying that $g(x)=f(x)+6$. So if you want to type in that equation into Desmos for g, Desmos will actually graph g for us. Either method will show us that our answer is 21.

14. **C**

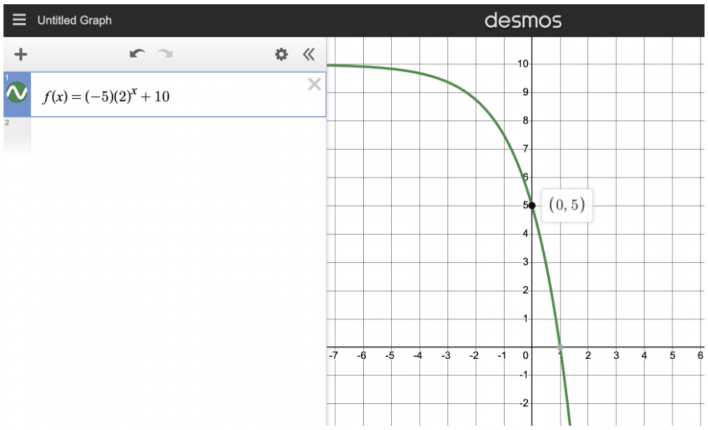
The graph has x-intercepts at 5 and -1, so the answer has to be either C or D. And since the graph touches and turns back at -1, the multiplicity of this root is 2, meaning we need $(x+1)^2$ in our answer. So we get C.

Finding Intercepts (Answers)

1. **A**

Here's the key for this problem: *don't panic!* Just because you probably don't know what the heck the graph will look like doesn't matter at all. Confidently go to Desmos and type in the equation exactly as you see it and just find the y-intercept. It's really that simple!

We see from the graph in Desmos that the right answer is A: $(0, 5)$.



<u>Bonus math</u>: you don't really *need* to know the math behind this one to get it right, but let's talk about it anyway: to find the y-intercept, we know we just plug in $x = 0$ into the function. So let's do that and see if we get the same answer:

$$\begin{aligned} f(x) &= (-5)(2)^x + 10 && \text{the original function} \\ &= (-5)(2)^0 + 10 && \text{plug in } x = 0 \\ &= (-5)(1) + 10 && 2^0 = 1 \\ &= -5 + 10 && \text{simplify} \\ &= 5 && \text{simplify} \end{aligned}$$

And this corresponds to the point $(0, 5)$, so we do, of course, get the same answer.

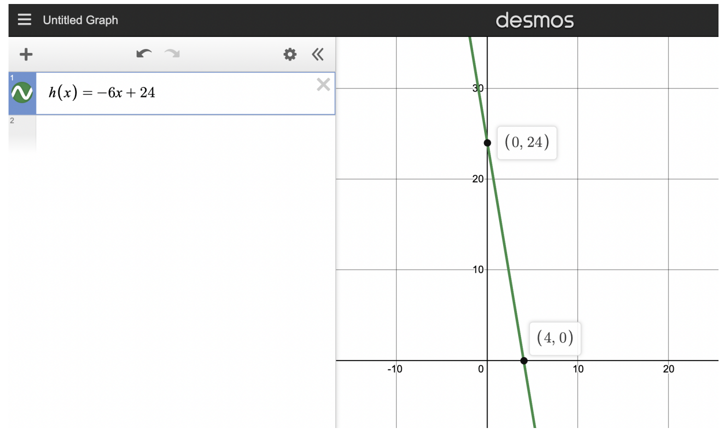
---

2. **4**

This is simple: the y-value where the graph crosses the y-axis is 4.

3. **28**

We can easily go to Desmos to find the values of the intercepts. Because a refers to the x-intercept, we can see that $a = 4$. Since b is the y-intercept, we can see that $b = 24$. Therefore, $a + b = 4 + 24 = 28$.



---

4. **-5**

$f(0)$ is another way of asking where the graph crosses the y-axis. That place clearly is -5, which is our answer.

---

5. **D**

Here we can either use Desmos to find the x-intercept or plug 0 in for $f(x)$ in the function—remember, at the x-intercept, y (or $f(x)$) is always 0:

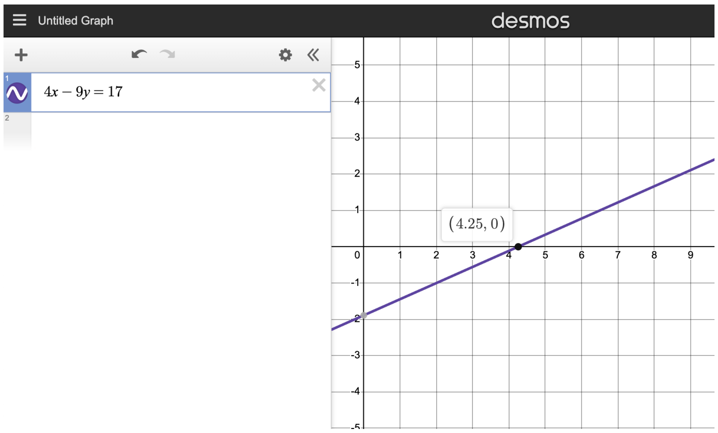
$$\begin{aligned} f(x) &= 2x - 38 && \text{the original function} \\ 0 &= 2x - 38 && \text{substitute 0 in place of } f(x) \\ 38 &= 2x && \text{isolate the } x \text{ term} \\ x &= 19 && \text{divide by 2} \end{aligned}$$

This is where the graph crosses the x-axis, which corresponds to the point $(19, 0)$.

6. **21/4** or **5.25**

Maybe this is a good time to discuss what happens algebraically to the equation of a function when a translation (a shift) is applied to it. If you've learned this before, you probably remember that a horizontal shift has to do with the x in the equation and a vertical shift has to do with the y. Let's just choose a simple number like 5 to serve as an example:

$$\boxed{\begin{array}{c}\text{To move a graph } \underline{\text{left}} \text{ } 5\text{, we replace } x \text{ in the equation with } (x+5)\\ \text{To move a graph } \underline{\text{right}} \text{ } 5\text{, we replace } x \text{ in the equation with } (x-5)\\ \text{To move a graph } \underline{\text{down}} \text{ } 5\text{, we replace } y \text{ in the equation with } (y+5)\\ \text{To move a graph } \underline{\text{up}} \text{ } 5\text{, we replace } y \text{ in the equation with } (y-5)\end{array}}$$



We should go to Desmos and first see what the original graph's x-intercept is, and from there, we could just add 1 to this value to get our answer for the new graph's x-intercept.

Since the original intercept is 4.25, the shifted graph's intercept will be 5.25.

7. **17/7** or **2.43**

This one falls into the harder category since we have a horizontal shift but are asked about the y-intercept.

We could definitely go to Desmos here, but what will we need to look for? Remember that a shift left 6 units corresponds to changing the x in the equation to $(x+6)$. Doing that gives us a new equation: $3(x+6)+7y=35$.

Now we can type this into Desmos to see its y-intercept, but like we saw in an earlier example, this decimal is messy and we'll want to find its exact value, so let's use algebra. We plug in $x=0$ because we're finding the y-intercept:

$$3(x+6)+7y=35$$

$$3(0+6)+7y=35$$

$$3(6)+7y=35$$

$$18+7y=35$$

$$7y=17$$

$$\boxed{y=\frac{17}{7}}$$

8. **14**

The points $\left(\frac{17}{2}, 0\right)$ and $\left(0, -\frac{51}{8}\right)$ are the intercepts of the linear function $ax+by=51$ in the xy-plane, where a and b are integers. What is the value of $a-b$?

We could use each point that's given to us to identify the coefficients a and b. Here's how: since $\left(\frac{17}{2}, 0\right)$ is on the line, we could plug $x=\frac{17}{2}$ and $y=0$ together into the equation $ax+by=51$:

$$\begin{array}{rc}\frac{17}{2}a+0b=51 & \text{plug in our } x \text{ and } y\\ \frac{17}{2}a=51 & \text{simplify}\\ a=51\cdot\frac{2}{17} & \text{divide (multiply by the reciprocal)}\\ a=6 & \text{simplify}\end{array}$$

Similarly, since $\left(0, -\frac{51}{8}\right)$ is on the line, we could plug $x=0$ and $y=-\frac{51}{8}$ together into that same equation, $ax+by=51$:

$$\begin{array}{rc}0a-\frac{51}{8}b=51 & \text{plug in our } x \text{ and } y\\ -\frac{51}{8}b=51 & \text{simplify}\\ b=51\left(-\frac{8}{51}\right) & \text{divide (multiply by the reciprocal)}\\ a=-8 & \text{simplify}\end{array}$$

Finally, let's find $a-b$:

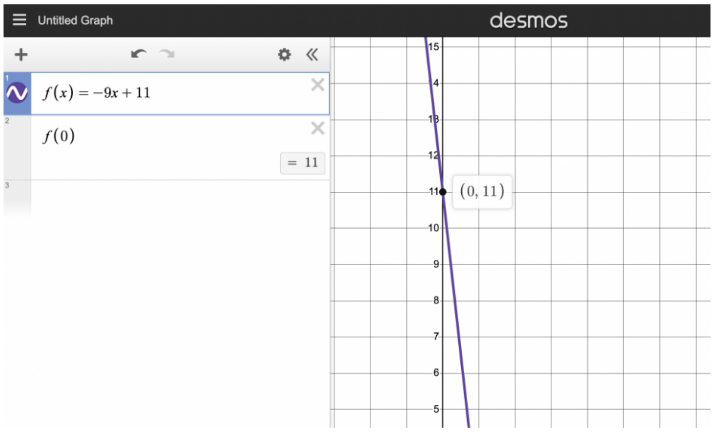
$$\begin{aligned}&a-b\\ &=6-(-8)\\ &=14\end{aligned}$$

Be careful of the double negative!

9. **7**

Different wording, but the same type of problem: tell them where the graph crosses the y-axis. It's clear that this is at 7.

10. **11**



We learned that $f(0)$ is another way of asking for the y-intercept. To find it, we can either plug in 0 for x in the function, or we can type the function into Desmos and see where it crosses the y-axis. Either way, the answer is 11.

Desmos bonus tip: notice that if you type the function into Desmos and then type in $f(0)$ in the second box, Desmos automatically tells you that this has a value of 11!

---

11. **B**

To find both intercepts, we could go to Desmos, but this is another one of those problems in which both intercepts are messy decimals and the answer we get from using them won't be exact enough. So our best bet is to do this out algebraically.

Let's plug in $x = 0$ to find the y-intercept (b) and then we'll plug in $y = 0$ to find the x-intercept (a):

y-intercept $(x = 0)$	x-intercept $(y = 0)$
$-11x + 3y = 13$	$-11x + 3y = 13$
$-11(0) + 3y = 13$	$-11x + 3(0) = 13$
$3y = 13$	$-11x = 13$
$y = \frac{13}{3}$	$x = -\frac{13}{11}$

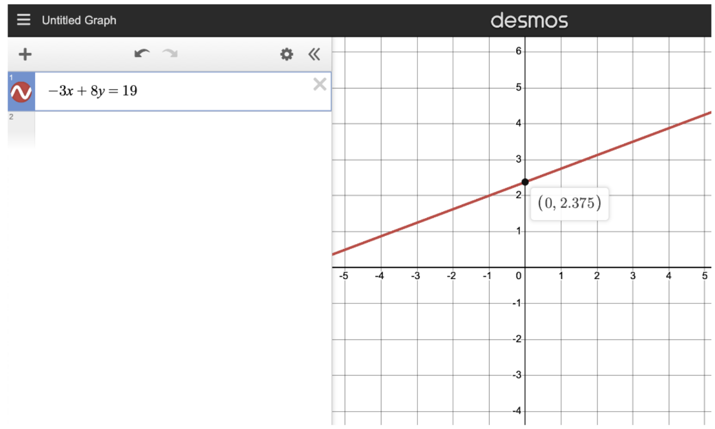
This means $a = -\frac{13}{11}$ and $b = \frac{13}{3}$. Now we just have to divide them to find $\frac{a}{b}$ (and feel free to do this on the calculator if you'd like):

$$\frac{a}{b} = \frac{-\frac{13}{11}}{\frac{13}{3}}$$

$$= -\frac{13}{11} \cdot \frac{3}{13} \qquad \text{dividing by a fraction} = \text{multiplying by its reciprocal}$$

$$\boxed{= -\frac{3}{11}}$$

12. **⅜ or .375**



We could use Desmos for this one, and when we do, we could just take the y-intercept of the original equation, 2.375 (an exact decimal, so we can do it this way), and subtract 2, since the shift is in the y (up and down) direction:

$$2.375 - 2 = 0.375$$

You can either use this decimal as your answer or the equivalent fraction, which is $\frac{3}{8}$.

If you did it the algebraic way, you'd have switched out y for $(y + 2)$ because of the shift down 2, then you'd have plugged $x = 0$ into the new equation to find the y-intercept:

$$-3x + 8(y + 2) = 19$$

$$-3(0) + 8(y + 2) = 19$$

$$8(y + 2) = 19$$

$$8y + 16 = 19$$

$$8y = 3$$

$$y = \frac{3}{8}$$

---

13. **D**

What this comes down to is finding the y-intercept, which they call n. One thing that's easy to miss is that the y-intercept is $(m + 1, n)$. Well, remember, any y-intercept's x-value is 0, so this tells us that $m + 1 = 0$, which means that $m = -1$.

And now we know the exact coordinates of both of the other points on this line:

x	y
m	14
$m + 8$	-10

$\rightarrow$

x	y
-1	14
7	-10

solution is continued on the next page...

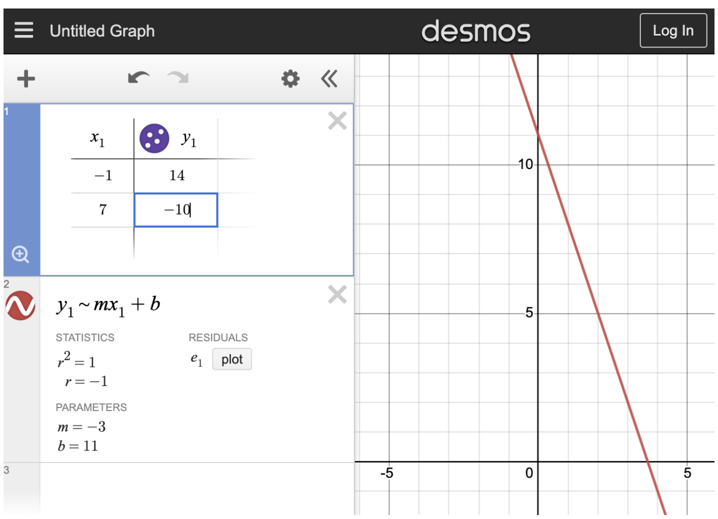
Of course, these are the points $(-1, 14)$ and $(7, -10)$, both of which we can plot using Desmos. The line that goes through both of these points will have a certain y-intercept, and it's that value that we need to find.

Let's try to find the equation of this line in slope-intercept form, $y = mx + b$, because if we figure out this equation, our answer is just the value of b.

Desmos can actually do this for us! Click the plus sign $(+)$ on the top-left of the screen in Desmos, and then select "table." Type in the numbers in our table, as we see here:

x	y
-1	14
7	-10

On a new line in Desmos, type in $y_1 \sim mx_1 + b$ (to get the little 1's under the variables, hold the shift key and hit the minus sign. But then make sure you click your keyboard arrow to the right to get Desmos to type the next part back at the normal height! For the $\sim$ symbol, hold the shift key and hit the key to the left of the 1).



At the very bottom of that box, Desmos tells us that the value of m is -3 and the value of b is 11, which is our answer.

Evaluating Functions at a Given Value (Answers)

1. **36**

They want us to plug 3 into the function for x to find y. This gives us

$$y = 4x^2$$
$$y = 4(3)^2$$
$$y = 4(9)$$
$$\boxed{y = 36}$$

2. **A**

This time, they tell us the value of the function, 25, and we need to determine which x-value was plugged in to produce that value. So we can just plug in 25 in place of $f(x)$ and solve for x:

$$f(x) = 11x + 3$$
$$25 = 11x + 3$$
$$22 = 11x$$
$$\boxed{x = 2}$$

3. **B**

Let's evaluate the function $f(x) = 5x - 11$ at each of the three x-values given in each table to see which table's points work:

$x = 0$	$x = 2$	$x = 4$
$f(x) = 5x - 11$	$f(x) = 5x - 11$	$f(x) = 5x - 11$
$f(0) = 5(0) - 11$	$f(2) = 5(2) - 11$	$f(4) = 5(4) - 11$
$= 0 - 11$	$= 10 - 11$	$y = 20 - 11$
$= -11$	$= -1$	$= 9$

These results match up with the table in B, which is our answer.

The alternative way to solve this is to put the function $f(x) = 5x - 11$ right into Desmos to see which of the tables' three points are all on the graph of the function. Remember, $f(x)$ is the same thing as y, so each of the tables really gives us three (x, y) coordinates. The only table that has all three of its coordinate pairs on the graph of $f(x) = 5x - 11$ is the one in B.

4. **D**

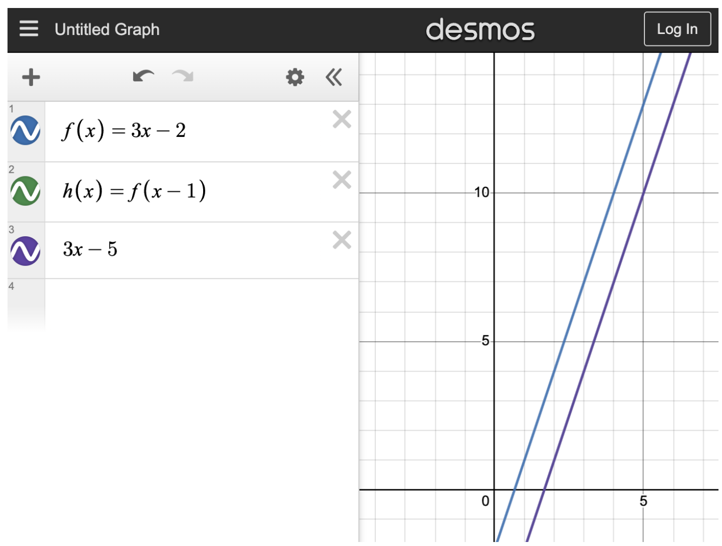
Since $h(x) = f(x - 1)$, this means that we can plug in $x - 1$ in place of x in the function $f(x)$ to find $h(x)$:

$$f(x) = 3x - 2$$
$$h(x) = 3(x - 1) - 2$$
$$= 3x - 3 - 2$$
$$= 3x - 5$$

So the answer is D.

Be careful! It's easy to forget that we need to multiply 3 by *the entire expression* $x - 1$, not just by the x. If you got A as your answer, you probably didn't distribute the 3 as we just described.

There's another way we can do this in Desmos: type in the two functions exactly as they're given to us, $f(x) = 3x - 2$ and $h(x) = f(x - 1)$. Notice which of them is $h(x)$ on the graph. Now, we can test out each of the four answers by typing it in and seeing if its graph overlaps with the graph of $h(x)$. Only one will, and that will be our answer. It ends up being D, $3x - 5$.



5. **D**

Since the time is our variable t, and the time they ask about is 5 seconds, we know our t-value is 5. And since they ask about the distance, and this is our d, it should be clear that we just need to plug in $t = 5$ into our function to find out what d is:

$$d = 25t$$
$$d = 25(5)$$
$$\boxed{d = 125}$$

6. **5**

The number of infants is 12, and this corresponds to x, so $x = 12$. They want us to find the number of toddlers, which is y, so we need to plug $x = 12$ into our equation and solve it for y:

$$\begin{aligned} 3.5x + 2y &= 52 && \text{the original function} \\ 3.5(12) + 2y &= 52 && \text{plug in 12 for } x \\ 42 + 2y &= 52 && \text{multiply} \\ 2y &= 10 && \text{isolate } 2y \\ y &= 5 && \text{divide by 2} \end{aligned}$$

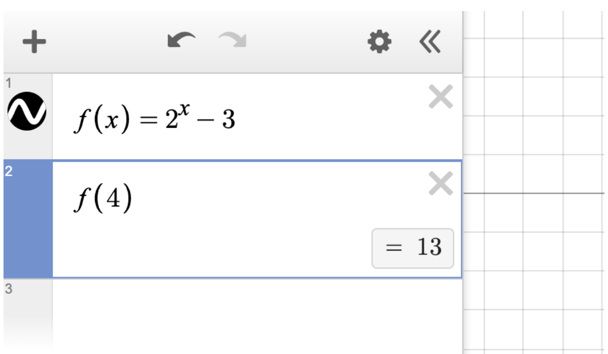
So the number of toddlers is 5.

7. **13**

Let's just plug in 4 in place of x in the given function:

$$\begin{aligned} f(x) &= 2^x - 3 \\ f(4) &= 2^4 - 3 \\ &= 16 - 3 \\ &= 13 \end{aligned}$$

Desmos can also evaluate this for you: type in the function exactly as it's given to us and then type in f(4) and you'll automatically see the answer, 13:



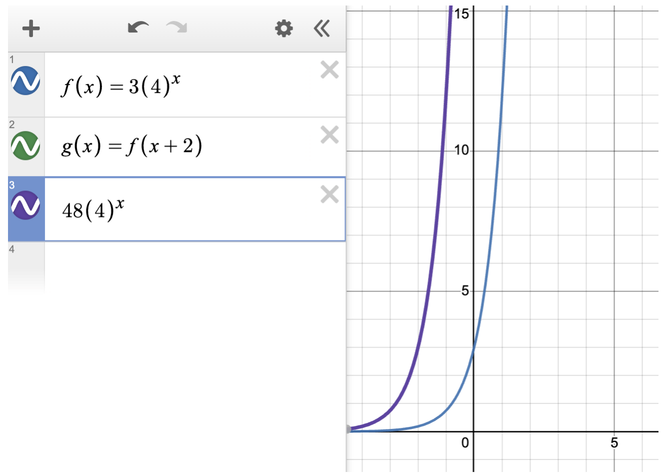
8. **C**

We're told that $f(x) = -13$, so we can take the function $f(x) = 11 - 3x$ and plug in -13 in place of $f(x)$, solving the equation for x:

$$\begin{aligned} f(x) &= 11 - 3x \\ -13 &= 11 - 3x \\ -24 &= -3x \\ x &= 8 \end{aligned}$$

9. **D**

What's a quick way to get to the answer? Try Desmos. Type in the functions $f(x) = 3(4)^x$ and $g(x) = f(x+2)$. Even though this doesn't tell us our answer right away, we can test each of the four answers by typing them into Desmos and seeing which one's graph matches up with the graph of $g(x) = f(x+2)$. This will confirm that D is our answer.



10. **A**

Notice that each of the tables contains the same three x-values: $-1, 0$, and 1. We can *evaluate* the function $f(x) = 2x^2 + 9$ at each of these three x-values to see which table's points work:

$$\begin{aligned} x &= -1 \\ f(x) &= 2x^2 + 9 \\ f(x) &= 2(-1)^2 + 9 \\ &= 2(1) + 9 \\ &= 11 \end{aligned} \qquad \begin{aligned} x &= 0 \\ f(x) &= 2x^2 + 9 \\ f(x) &= 2(0)^2 + 9 \\ &= 2(0) + 9 \\ &= 9 \end{aligned} \qquad \begin{aligned} x &= 1 \\ f(x) &= 2x^2 + 9 \\ f(x) &= 2(1)^2 + 9 \\ y &= 2(1) + 9 \\ &= 11 \end{aligned}$$

These results match up with the table in A, which is our answer.

Plugging the function into Desmos would also show us that A's three points are all on the graph of the given function.

11. **280**

Simply plug 75 into the function in place of x since this variable represents our temperature:

$$\begin{aligned} f(x) &= 8x - 320 \\ f(75) &= 8(75) - 320 \\ &= 600 - 320 \\ &\boxed{= 280} \end{aligned}$$